



MATHEMATICS LESSON

GRADE 7



DATE:

TOPIC: GEOMETRIC AND NUMERIC PATTERNS:

<p>CONCEPTS & SKILLS TO BE ACHIEVED: By the end of the lesson learners should know and be able to:</p> <ul style="list-style-type: none"> • create, recognise, describe, extend and make generalisations • make predictions. • work with different representations, such as flow diagrams, tables and graphs. 	
<p>RESOURCES:</p>	<p>DBE Workbook 1, Sasol-Inzalo book, Textbooks</p>

DAY 1	
NUMERIC PATTERNS	
INTRODUCTION	
<p>REVISION ACTIVITY: <i>Ensure that you know the correct terminology</i> consecutive, terms, number sequence</p>	<p>DEFINITIONS:</p>
<p>What comes next?</p> <p>What may the next three numbers in each of these sequences be?</p> <p>4; 8; 12; 16; 20; _____</p> <p>4; 8; 16; 32; 64; _____</p> <p>4; 8; 14; 22; 32; _____</p> <p>5; 7; 4; 8; 3; 9; 2; _____</p>	<p>The numbers in a sequence are called the terms of the sequence. Terms that follow one another are said to be consecutive.</p>
<p>A set of numbers in a given order is called a number sequence. In some cases, each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a pattern in the sequence.</p>	

LESSON DEVELOPMENT

Work through the following examples, and note how the development of the numeric patterns.



1. (a) Write down the next three numbers in each of these sequences:

Sequence A: 4; 7; 10; 13; 16; _____

Sequence B: 5; 10; 20; 40; 80; _____

Sequence C: 2; 5; 10; 17; 26; _____

(b) Write down how you decided what the next numbers would be in each of the three sequences.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

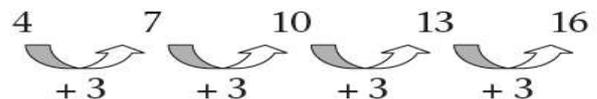
A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

In sequence A of question 1 there is a **constant difference** between consecutive terms, as shown below.

Sequence A: 4 7 10 13 16

Difference: + 3 + 3 + 3 + 3



In sequence B of question 1 there is a **constant ratio** between consecutive terms, as shown below.

Sequence B: 5 10 20 40 80

Ratio: $\times 2 \times 2 \times 2 \times 2$

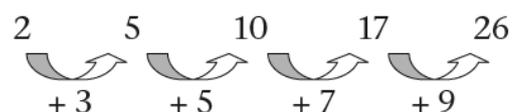


In sequence C of question 1 there is neither a constant difference nor a constant ratio between consecutive terms. There is, however, a pattern in the differences between the terms, which makes it possible to extend the sequence. Consecutive odd numbers,

starting with 3, are added to form the next term.

Sequence C: 2 5 10 17 26

Difference: + 3 + 5 + 7 + 9



CLASSWORK: ACTIVITY: Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum



WE ARE GOING TO PRACTICE THE DIFFERENT SKILLS.

1. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.

(a) 100; 95; 90; 85; _____

(b) 0,3; 0,5; 0,7; 0,9; _____

(c) 6; 18; 54; 162; _____

(d) 1; 3; 6; 10; 15; _____

(e) 20; 31; 42; 53; _____

(f) 10; 9,7; 9,4; 9,1; _____

(g) 18 000; 1 800; 180; 18; _____

(h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{6}$; _____

(i) 1; 4; 9; 16; _____

(j) 625; 125; 25; 5; _____

The word “recur” means “to happen again”. The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**

CONSOLIDATION

YOU SHOULD REMEMBER FROM TODAY’S WORK THAT:

- **To use the correct vocabulary is important.**
- A **sequence** is a set of numbers in a given order.
- In a sequence each number can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence.
- The numbers in a sequence are called the **terms** of the sequence.
- Terms that follow one another are said to be **consecutive**.
- A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.
- A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.
- A sequence can also be formed in such a way that **neither** the difference **nor** the ratio between one term and the next is constant.

HOMEWORK: Complete the activity in your classwork book.

1. Find the missing terms in each of the following sequences. Write the rule for each of the number patterns below.

(a) 5; 20; 80; 320; _____; _____; _____

(b) 1; 3; 9; 27; _____; _____; _____

(c) 3200; 1600; 800; _____; _____; _____

(d) 15; _____; 60; 120; _____; _____; 960

(e) 41; 4,1; 0,41; 0,041; _____; _____; _____

2. Give the rule to describe the relationship between the numbers in the sequences below. Use the rule to give the next three numbers in the sequence:

(a) 3; 7; 11; 15; ____; ____; ____

(b) 120; 115; 110; 105; ____; ____

(c) 2; 4; 8; 16; ____; ____; ____

(d) 1; 2; 4; 7; 11; 17; ____; ____; ____

DAY 2

LESSON DEVELOPMENT

INTRODUCTION

REVISION ACTIVITY:

Ensure that you know the correct terminology

dependent variable, independent variable



DEFINITIONS:

Relationships Between Dependent and Independent Variables

1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Complete the table below to show how much his parking costs him per day for various numbers of hours.

# hours	1	2	3	4	5	6	7	8	9
Cost of parking in R	5	7	9						

(b) How did you complete this table? Describe your method.

(c) Is there another way that you could complete the table? Describe it.

(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Complete the flow diagram to show Thembi's rule:



CLASSWORK: Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum

WE ARE GOING TO PRACTICE THE DIFFERENT SKILLS.

Answer the following questions.

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

1. (a) Complete the above table.
- (b) How did you calculate term number 50?
- (c) Lungile reasoned like this:

I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

Complete the pattern below to show Lungile's thinking:

Term 1: $11 + 1 \times 4 = 11 + 4 = 15$

Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

Term 3: _____

Term 4: _____

Term 5: _____

Term 6: _____

Term 10: _____

Term 50: _____

Lungile remembered that multiplication is done before addition, unless otherwise indicated by brackets.

- (d) Describe in your own words how term number 50 can be calculated.

CONSOLIDATION: YOU SHOULD REMEMBER FROM TODAY'S WORK THAT:

- The rule *multiply by 2 and then add 3* describes the relationship between the two variables in this situation.
- The number of hours is the **independent variable**.
- The cost of Mr Twala's parking is the **dependent variable** because the amount he has to pay *depends on* the number of hours that he parks.
- This rule describes how you can calculate the value of the *dependent* variable if the corresponding value of the *independent* variable is known. It differs from a recursive rule, which describes how you can calculate the value of the *dependent* variable that follows on a given value of the *dependent* variable.
- In the case of a number sequence, the **position** (number) of the term can be taken as the independent variable, as shown for the sequence 15; 19; 23; 27; 31; . . . in this table:

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

The R3 that is added is a **constant** in this situation. The number of hours and the cost are **variables**.

HOMEWORK: Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum

Activity

(a) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the 50th term. So, $15 + 49 \times 4 = 15 + 196 = 211$.*
Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15

Term 2: $15 + 1 \times 4 = 15 + 4 = 19$

Term 3: $15 + 2 \times 4 = 15 + 8 = 23$

Term 4: _____

Term 5: _____

Term 6: _____

Term 10: _____

Term 50: _____

(b) Write the rule to calculate term number 50 in your own words.

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

2. Write a rule to calculate the term for any term number in the sequence
15; 19; 23; 27; 31; . . . by using

(a) Lungile's thinking.

(b) Tilly's thinking.

We can use n as a symbol for "any term number".

The rule to calculate the term for any term number

when using Lungile's thinking will then be: Term = $n \times 4 + 11$

(c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking

DAY 3

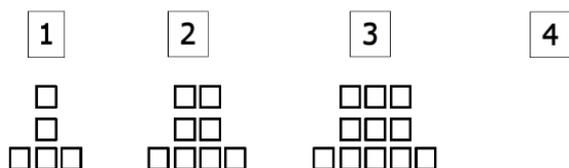
GEOMETRIC PATTERNS

LESSON DEVELOPMENT

CONSTANT QUANTITIES AND VARIABLE QUANTITIES

INTRODUCTION

Example 1: Study the figures below and answer the questions



Answer the questions below:

1. **WHAT DO WE NOTICE?** Squares were used to make the pattern.
2. **CAN YOU PREDICT HOW MANY SQUARES IN FIGURE 4?** 14
3. **DESCRIBE VERBALLY.** It is an increasing pattern.
4. **DESCRIBE IN WORDS.** 3 squares are being added to the next figure.
5. **REPRESENT IN A TABLE**

Terms	1	2	3	4	10
# squares	5	8	11	14	32

HOW DOES THE PATTERN DEVELOP?

i.e. How the pattern changes from one figure to the next. When you look carefully three squares are added in the 'middle' to the next figure so this can be seen as the **variable**. In the last layer the two squares, one on each end remains the stay, this can be seen as the **constant**.

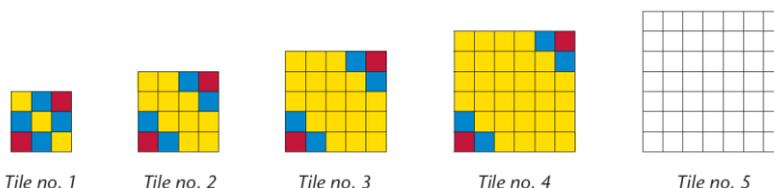
6. **DEVELOP A CALCULATION PLAN.**

Terms (n)	1	2	3	4	10
# squares	5	8	11	14	32
Calculations	$(1 \times 3) + 2$	$(2 \times 3) + 2$	$(3 \times 3) + 2$	$(2 \times 3) + 2$	$(10 \times 3) + 2$

CLASSWORK: ACTIVITY:

Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum

Small yellow, blue and red tiles are combined to form larger square tiles as shown below:



1. Draw tile no. 5 on the grid provided.
(Shade the blue and red tiles in different ways. You don't have to use colours.)

2. Complete the table

Tiles	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
# of yellow	3					
# of red	2					
# of blue	4					

- How many red tiles are there in each bigger tile?
- How many yellow tiles are there in each bigger tile?
- Some of the quantities in this situation are variables and some are constants.
Which are variables and which are constants?
- Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

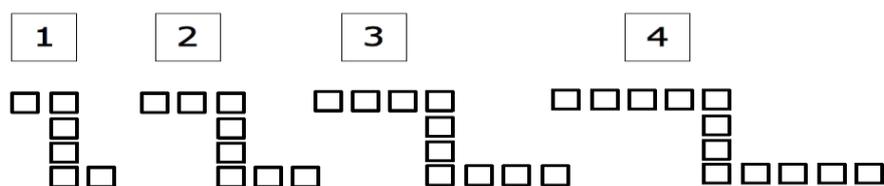
CONSOLIDATION: YOU SHOULD REMEMBER FROM TODAY'S WORK THAT:

- The number of red tiles is constant and the number of blue tiles is constant.
- It is clear that the design is such that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each.
- So the number of red and blue tiles is **constant** in this situation.
- The number of yellow tiles in the arrangements varies. The number of yellow tiles is a **variable** in this situation.

HOMEWORK

Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum.

Study the following pattern and answer the questions



- Describe the pattern
- Explain in your own words how the pattern develops or how the pattern grows from one figure to the next.
- Develop a calculation plan.
- Some of the quantities in this situation are variables and some are constants.
Which are variables and which are constants? Shade the variables and the constants in each figure.

DAY 4

LESSON DEVELOPMENT

PATTERNS WITH MATCHES

Question 1

A pattern with matches is shown below:



Figure 1



Figure 2



Figure 3

- (a) Explain how the pattern is formed.
 (b) Complete the table.

Figure number (n)	1	2	3	4	5	6	7	8
Number of matches	3	5	7					

- (c) What rule did you use to complete the table?
 (d) How many matches are needed to form figure 9?
 (e) How many matches are needed to form figure 17? Explain.
 (f) If you used the recursive rule to complete the table, it would have taken a long time to answer question (e) because you had to add the same number many times. Try to find an easier way to answer question (e). Describe your method.
 (g) Complete the pattern below.

Term 1: $1 + 1 \times 2 = 3$

Term 2: $1 + 2 \times 2 = 5$

Term 3: $1 + 3 \times 2 = 7$

Term 4: _____

Term 5: _____

Term 10: _____

Term 17: _____

Hint: It may help to think of figure no. 1 or term 1 like this:
 There is 1 match at the beginning and two more are added every time. It helps to “see” the two matches that are added each time.

- (h) What stays the same in the pattern in (g) and what varies?
 (i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure #



- (j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

CONSOLIDATION: YOU SHOULD REMEMBER FROM TODAY'S WORK THAT:

- The rule can be used to calculate any number of objects in the pattern.
- A rule can be recursive.
- There are constants and variables in a pattern.
- The rule can be used in a flow diagram to calculate the number of objects in any figure in a pattern.

The rule that describes the relationship between consecutive terms, is called a **recursive rule**.

CLASSWORK

Question 2

Another pattern with matches is shown below.

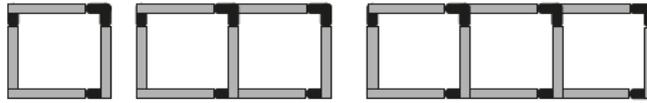


Figure 1

Figure 2

Figure 3

- (a) Explain how the pattern is formed.
 (b) Complete the table.

Figure no.	1	2	3	4	5	6	7	8	9
# Matches	4								

- (c) What rule did you use to complete the table?
 (d) How many matches are needed for figure 9 (or term 9)?
 (e) How many matches are needed for figure 20 (or term 20)?
 (f) What rule did you use to calculate the number of matches in question (e)?
 (g) Complete the pattern:
 Term 1: $1 + 1 \times 3 = 4$
 Term 2: $1 + 2 \times 3 = 7$
 Term 3: $1 + 3 \times 3 =$
 Term 4: _____
 Term 5: _____
 Term 10: _____
 Term 17: _____

- (h) What stays the same in the pattern in (g) and what varies?
 (i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

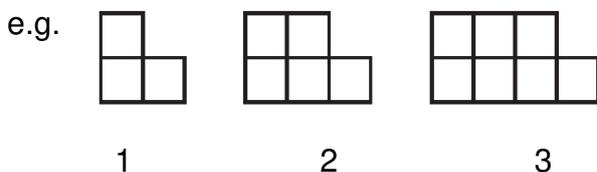


3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

Create your own pattern.

Create your own pattern using the rule: $n \times 2 + 1$.

Sketch the first three terms of a pattern and complete the table.



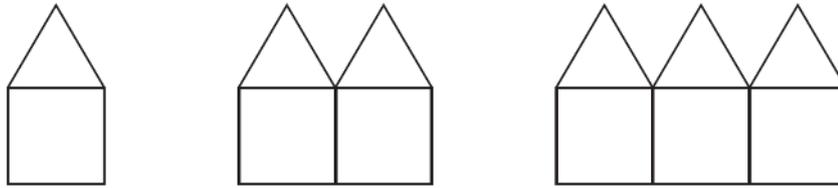
Terms	1	2	3
# Squares	3	5	7
Calculation	$1 \times 2 + 1$	$2 \times 2 + 1$	$3 \times 2 + 1$

4. Sketch the first three terms of a pattern that can be represented by $n^2 + 2n$

HOMEWORK

Question 3

Ralph built houses with match sticks. It looks like the figures below.



- (a) Explain how the pattern is formed.
(b) Complete the table.

Figure no.	1	2	3	4	5	6	7	8	9
# Matches	6	11	16						

- (c) What rule did you use to complete the table?
(d) How many matches are needed for figure 9 (or term 9)?
(e) How many matches are needed for figure 20 (or term 20)?
(f) What rule did you use to calculate the number of matches in question (e)?
(g) Complete the pattern:

Term 1: $1 \times 5 + 1$

Term 2: $2 \times 5 + 1$

Term 3: $3 \times 5 + 1$

Term 4: _____

Term 5: _____

Term 10: _____

Term 17: _____

- (h) What stays the same in the pattern in (g) and what varies?

- (i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.



DAY 5

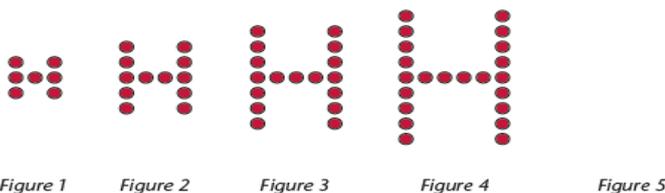
ALPHABETIC PATTERNS: Other objects can be used to form patterns too.

LESSON DEVELOPMENT

What have we learnt so far:

- So far, we have calculated the rule of a pattern.
- We can determine the number of objects in a picture of any number.
- We can determine difference between a preceding term and the following term in a number sequence.
- We can create our own pattern.

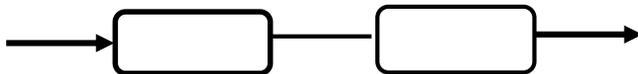
Consider the figures below formed with red dots.



1. How many dots are used to form figure 5?
2. Draw figure 5.
3. Complete the table.

figure	1	2	3	4	5	6	7	8	9
# dots	7								

4. Complete the flow diagram.



5. Which rule did you use to complete the table? Describe your rule.
6. Can you think of another rule to complete the table? Describe your rule.
7. Name the dependent variable and the independent variable in this situation.

CLASSROOM ACTIVITY: Complete the activities in your classwork book. First attempt yourself, before finding the answers in the Memorandum.

SQUARES AND CUBES

1. Squares are arranged to form figures as shown below.



- (a) Complete the table.

Figure	1	2	3	4	5	6	7	8	9
# squares	2	5	7	9					

- (b) Describe the recursive rule that you can use to extend the pattern in words.
- (c) Nombuso played around with the differences between consecutive terms. She noticed that the pattern (+ 3; + 5; + 7; ...) was similar to the one that you get when you calculate the differences between square numbers. This made her think that she should investigate square numbers to help her find a rule that could link the figure number and the number of squares.

Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $(1 \times 1) + 1 = 1 + 1 = 2$

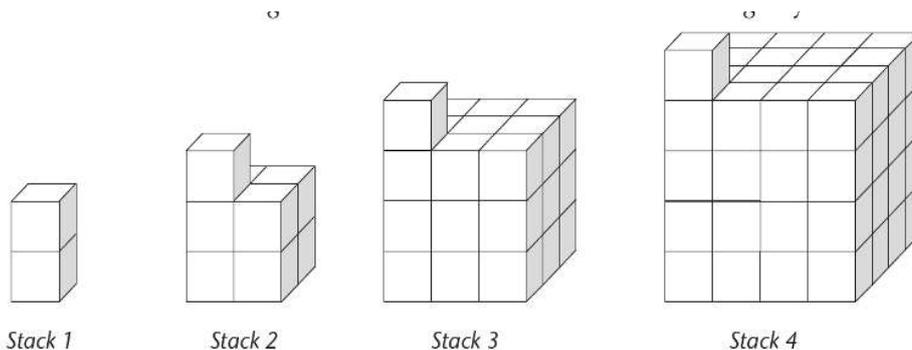
Figure 2: $(2 \times 2) + 1 = 4 + 1 = 5$

Etc. Figure 50:

- (d) Write a rule to calculate the number of squares for any figure.
- (f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

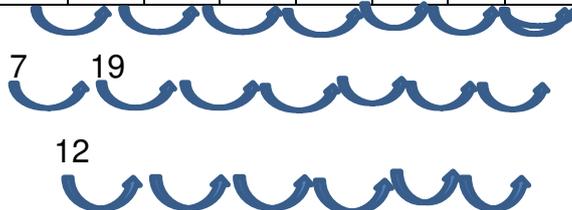
HOMEWORK

1. Identical cubes are arranged to form stacks of cubes in the following way:



- (a) Complete the table. Then find the difference between consecutive terms. Do it a second and a third time and fill in the differences in the spaces provided.

Stack (n)	1	2	3	4	5	6	7	8	9
# cubes	2	9	28						



- (b) Describe the way in which you completed the table.

- (c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$

Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$

Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$

Stack 5:

Stack 6:

Stack 7:

Stack 8:

Stack 9:

Stack 10:

- (d) How many cubes will there be in stack 50?
(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.
(f) Write your rule in (e) in terms of n where n is the symbol for any stack number.

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.

- (a) What did you find when you kept on finding the differences, as suggested in question 2(a)?
(b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

MEMORANDUM: DAY 1

INTRODUCTION

What comes next?

What may the next three numbers in each of these sequences be?

4; 8; 12; 16; 20; **24; 28; 32**

4; 8; 16; 32; 64; **128; 256; 512**

4; 8; 14; 22; 32; **44; 58; 74**

5; 7; 4; 8; 3; 9; 2; **10; 1; 11**

LESSON DEVELOPMENT

1. (a) Write down the next three numbers in each of these sequences:

Sequence A: 4; 7; 10; 13; 16; **19; 22; 25**

Sequence B: 5; 10; 20; 40; 80; **160; 320; 640**

Sequence C: 2; 5; 10; 17; 26; **37; 50; 65**

(b) Write down how you decided what the next numbers would be in each of the three sequences.

A: Add 3 to the previous term.

B: Multiply the previous term by 2.

C: Add 2 more to the previous term than you added to the one before that.

CLASSWORK:

2. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.

(a) 100; 95; 90; 85; **80; 75; 70; 65; 60**

Each term is 5 less than the previous term.

(b) 0,3; 0,5; 0,7; 0,9; **1,1; 1,3; 1,5; 1,7; 1,9**

Each term is 0,2 more than the previous term.

(c) 6; 18; 54; 162; **486; 1 458; 4 374; 13 122; 39 366**

Each term is 3 times the previous term.

(d) 1; 3; 6; 10; 15; **21; 28; 36; 45; 55**

The difference between the first two terms is 2. Thereafter, the difference is 1 more than the previous difference in each case. OR: Add 2 to the first term, then add one more than previously to get the next term in each case.

(e) 20; 31; 42; 53; **64; 75; 86; 97; 108**

Each term is 11 more than the previous term.

(f) 10; 9,7; 9,4; 9,1; **8,8; 8,5; 8,2; 7,9; 7,6**

Each term is 0,3 less than the previous term.

(g) 18 000; 1 800; 180; 18; **1,8; 0,18; 0,018; 0,0018**

Each term is a tenth of the previous term.

(h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{6}$; $\frac{1}{3}$; $\frac{2}{3}$; $1\frac{1}{3}$; $2\frac{2}{3}$; $5\frac{1}{3}$

Each term is two times the previous term.

(i) 1; 4; 9; 16; 25; 36; 49; 64; 81

The differences between consecutive terms are the odd numbers starting at 3.

(j) 625; 125; 25; 5; 1; 0,2; 0,04; 0,008; 0,0016

Each term is a fifth of the previous term.

HOMEWORK:

1. (a) 5; 20; 80; 320; 1 280; 5 120; 20 480: Rule: $\times 4$

(b) 1; 3; 9; 27; 81; 243; 729: Rule: $\times 3$

(c) 3200; 1600; 800; 400; 200; 100: Rule: halving or $\div 2$

(d) 15; 30; 60; 120; 240; 480; 960: Rule: doubling or $\times 2$

(e) 41; 4,1; 0,41; 0,041; 0,0041; 0,00041; 0,000041: Rule: $\div 10$

2.

(a) 3; 7; 11; 15; 19; 23; 27: Rule: $+ 4$

(b) 120; 115; 110; 105; 100; 95: $- 5$

(c) 2; 4; 8; 16; 32; 64; 128: double the preceding term or $\times 2$

(d) 1; 2; 4; 7; 11; 16; 22; 29; 37: add consecutive numbers to preceding term.

MEMORANDUM: DAY 2

LESSON DEVELOPMENT

1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Complete the table below to show how much his parking costs him per day for various numbers of hours.

Figure	1	2	3	4	5	6	7	8	9
# tiles	5	8	11	14	17	20	23	26	29

(b) How did you complete this table? Describe your method.

"I added 2 to the previous term each time" is a likely and good answer.

(c) Is there another way that you could complete the table? Describe it.

Multiply the number of hours by 2 and then add 3.

(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Complete the flow diagram to show Thembi's rule.

Number of hours Cost in rands



Cost in rands

CLASSWORK

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31	35	39	43	211

1 (a) Complete the above table.

(b) How did you calculate term number 50?

Starting at 43 and adding 4 repeatedly 42 times is the most likely method, though some learners may do $43 + 42 \times 4 = 43 + 168$ or even, if they recognise the relationship, $50 \times 4 + 11$ (this would be very smart).

(c) Lungile reasoned like this:

I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

Complete the pattern below to show Lungile's thinking:

Term 1: $11 + 1 \times 4 = 11 + 4 = 15$

Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

Term 3: $11 + 3 \times 4 = 11 + 12 = 23$

Term 4: $11 + 4 \times 4 = 11 + 16 = 27$

Term 5: $11 + 5 \times 4 = 11 + 20 = 31$

Term 6: $11 + 6 \times 4 = 11 + 24 = 35$

Term 10: $11 + 10 \times 4 = 11 + 40 = 51$

Term 50: $11 + 50 \times 4 = 11 + 200 = 211$

(d) Describe in your own words how term number 50 can be calculated.

Add 49 times 4 to 15. OR: Multiply 50 by 4 and add the answer to 11.

HOMWORK

e) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the 50th term. So, $15 + 49 \times 4 = 15 + 196 = 211$.*

Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15

Term 2: $15 + 1 \times 4 = 15 + 4 = 19$

Term 3: $15 + 2 \times 4 = 15 + 8 = 23$

Term 4: $15 + 3 \times 4 = 15 + 12 = 27$

Term 5: $15 + 4 \times 4 = 15 + 16 = 31$

Term 6: $15 + 5 \times 4 = 15 + 20 = 35$

Term 10: $15 + 9 \times 4 = 15 + 36 = 51$

Term 50: $15 + 49 \times 4 = 15 + 196 = 211$

(f) Write the rule to calculate term number 50 in your own words.

Subtract 1 from the term number, multiply the result by 4, then add 15.

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

3. Write a rule to calculate the term for any term number in the sequence 15; 19; 23; 27; 31; . . . by using

(a) Lungile's thinking.

Multiply the term number by 4 and add to 11.

(b) Tilly's thinking.

Multiply one less than the term number by 4 and add to 15.

We can use n as a symbol for "any term number".

The rule to calculate the term for any term number when using Lungile's thinking will then be:

Term = $n \times 4 + 11$

(c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking.

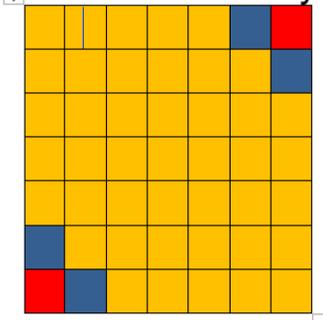
Term = $(n - 1) \times 4 + 15$

= $4(n - 1) + 15$

MEMORANDUM: DAY 3

CLASSWORK ACTIVITY

1. Draw tile no. 5 on the grid provided. (Shade the blue and red tiles in different ways. You don't have to use colours.)



2. Complete the table.

Tiles	1	2	3	4	5	10
# of yellow	3	10	19	30	43	138
# of red	2	2	2	2	2	2
# of blue	4	4	4	4	4	4

3. How many red tiles are there in each bigger tile? **2**

4. How many yellow tiles are there in each bigger tile?

The number grows. Start with 3 and add 7; then add two more each time.

OR: (The tile number plus two) squared minus 6.

5. Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?

The number of red tiles and the number of blue tiles are constants, while the number of yellow tiles and the tile number are variables.

6. Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

No, other patterns could also start with the same first tile.

HOMEWORK

1. **Increasing pattern**

2. **Draw the fifth and the sixth figures.**

3.

Terms (n)	1	2	3	4	5
# Squares	$2 \times 2 + 2$	$2 \times 3 + 2$	$2 \times 4 + 2$	$2 \times 5 + 2$	$2 \times 6 + 2$

4. **There are two squares in the middle of each figure; the two squares represent the constant. The tails increase by one on each side.**

MEMORANDUM DAY 4

LESSON DEVELOPMENT

Question 1.

(a) After the initial triangle, each further triangle uses two more matches.

(b) Complete the table.

Figure number (n)	1	2	3	4	5	6	7	8
Number of matches	3	5	7	9	11	13	15	17

(c) Add 2 to get the next term.

(d) 19

(e) 35. Multiply the figure number by 2, then add 1. OR:

Keep on adding 2 OR: Add 16 times 2 to 3.

(f) Multiply the figure number by 2, then add 1.

(g) Complete the pattern below.

Term 1: $1 + 1 \times 2 = 3$

Term 2: $1 + 2 \times 2 = 5$

Term 3: $1 + 3 \times 2 = 7$

Term 4: $1 + 4 \times 2 = 9$

Term 5: $1 + 5 \times 2 = 11$

Term 10: $1 + 10 \times 2 = 21$

Term 17: $1 + 17 \times 2 = 35$

(h) The 1 in the number expression and the 2 that the term number is multiplied by stay the same. The term (figure) number and the number of matches vary.

(i). *Figure number $\times 2 + 1$ Number of matches*

(j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

You add two matches each time, so that is the number that you multiply the figure number by in your rule.

CLASSWORK

Question 2

(a) Explain how the pattern is formed.

After the initial square, each further square uses three more matches.

(b)

Figure no.	1	2	3	4	5	6	7	8	9
# Matches	4	7	10	13	16	19	22	25	28

(c)

use to complete the table?

What rule did you

Add 3 to the number of matches in the preceding figure (or term).

(d) How many matches are needed for figure 9 (or term 9)? 28

(e) How many matches are needed for figure 20 (or term 20)? 61

(f) What rule did you use to calculate the number of matches in question (e)?

Multiply the term number by 3 and add 1.

(g) Complete the pattern:

Term 1: $1 + 1 \times 3 = 4$

Term 2: $1 + 2 \times 3 = 7$

Term 3: $1 + 3 \times 3 = 10$

Term 4: $1 + 4 \times 3 = 13$

Term 5: $1 + 5 \times 3 = 16$

Term 10: $1 + 10 \times 3 = 31$

Term 17: $1 + 17 \times 3 = 52$

(h) What stays the same in the pattern in (g) and what varies?

The 1 in each number expression and the 3 that the term is multiplied by stay the same. The figure number (or term) and the number of matches vary.

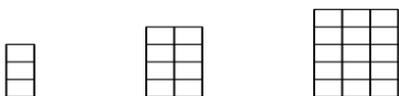
(i) Use the flow diagram below to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number $\times 3 + 1$ Number of matches

3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

In both cases, 1 is the added constant and the figure number is multiplied by a constant (changed from 2 to 3) to determine the number of matches.

4.



Term	1	2	3
# Squares	3	8	15
Calculation	$1 \times 1 + 2 \times 1$	$2 \times 2 + 2 \times 2$	$3 \times 3 + 2 \times 2$

HOMEWORK

Question 3

(a) After the initial pentagon each further pentagon was built by using five matches

(b)

Figure no.	1	2	3	4	5	6	7	8	9
# Matches	6	11	16	21	26	31	36	41	46

(c) add 5 matches to the preceding term.

(d) 46 matches

(e) $20 \times 5 + 1 = 101$

(f) $n \times 5 + 1$

(g) Term 1: $1 \times 5 + 1$

Term 2: $2 \times 5 + 1$

Term 3: $3 \times 5 + 1$

Term 4: $4 \times 5 + 1$

Term 5: $5 \times 5 + 1$

Term 10: $10 \times 5 + 1$

Term 17: $17 \times 5 + 1$

- (h) The 1 in each number expression and the 5 that the term is multiplied by stay the same. The figure number (or term) and the number of matches vary.



MEMORANDUM DAY 5

LESSON DEVELOPMENT

- How many dots are used to form figure 5? **27**
- Draw figure 5.
- Complete the table.

figure	1	2	3	4	5	6	7	8	9
# dots	7	12	17	22	27	32	37	42	47

- Complete the flow diagram.
Figure number $\times 5 + 2$ Number of dots
- What rule did you use to complete the table? Describe your rule.
Add 5 to the preceding term.
- Can you think of another rule to complete the table? Describe your rule.
**Figure number $+ (\text{figure number} \times 2 + 1) \times 2$ [based on the H-shapes]
= Figure number $\times 5 + 2$**
- Name the dependent variable and the independent variable in this situation.
Independent variable: figure number
Dependent variable: number of dots

CLASSWORK

- (a) (a) Complete the table.

Figure	1	2	3	4	5	6	7	8	9
# squares	2	5	7	9	11	13	15	17	19

- (b) Describe the recursive rule that you can use to extend the pattern in words.
Add the next odd number, starting from 3.

- (c) Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$
 Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$
 Figure 3: $3 \times 3 + 1 = 9 + 1 = 10$
 Figure 4: $4 \times 4 + 1 = 16 + 1 = 17$
 Figure 5: $5 \times 5 + 1 = 25 + 1 = 26$
 Figure 6: $6 \times 6 + 1 = 36 + 1 = 37$
 Figure 7: $7 \times 7 + 1 = 49 + 1 = 50$
 Figure 8: $8 \times 8 + 1 = 64 + 1 = 65$
 Figure 50: $50 \times 50 + 1 = 2\,500 + 1 = 2\,501$

(d) Write a rule to calculate the number of squares for any figure number.

Square the figure number, then add 1.

(e) Write your rule in (d) in terms of n where n is the symbol for any figure number.

Number of squares = $n^2 + 1$

(f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

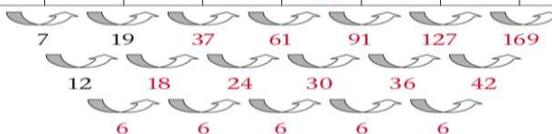
The letters H just used 5 more red dots each time, while the number of extra small squares needed was at first just 3, but increased each time you added the next odd number to the preceding term [+ 3; + 5; + 7; + 9; ...].

The letters H had a constant difference, while the difference between consecutive terms in the squares of this activity is not constant.

HOMEWORK

(a)

Stack number	1	2	3	4	5	6	7	8
Number of cubes	2	9	28	65	126	217	344	513



(b) _____ Describe the way in which you completed the table.

Learners' own answer, e.g. I noticed a pattern in the "differences" and followed that pattern; or: I cubed the stack number and added 1.

(c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

$$\text{Stack 1: } 1 \times 1 \times 1 + 1 = 1 + 1 = 2$$

$$\text{Stack 2: } 2 \times 2 \times 2 + 1 = 8 + 1 = 9$$

$$\text{Stack 3: } 3 \times 3 \times 3 + 1 = 27 + 1 = 28$$

$$\text{Stack 4: } 4 \times 4 \times 4 + 1 = 64 + 1 = 65$$

$$\text{Stack 5: } 5 \times 5 \times 5 + 1 = 125 + 1 = 126$$

$$\text{Stack 6: } 6 \times 6 \times 6 + 1 = 216 + 1 = 217$$

$$\text{Stack 7: } 7 \times 7 \times 7 + 1 = 343 + 1 = 344$$

$$\text{Stack 8: } 8 \times 8 \times 8 + 1 = 512 + 1 = 513$$

$$\text{Stack 9: } 9 \times 9 \times 9 + 1 = 729 + 1 = 730$$

$$\text{Stack 10: } 10 \times 10 \times 10 + 1 = 1\,000 + 1 = 1\,001$$

(d) How many cubes will there be in stack 50?

$$50^3 = 50 \times 50 \times 50 = 125\,000$$

$$125\,000 + 1 = 125\,001$$

(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.

Cube the stack number and add 1.

(f) Write your rule in (e) in terms of n where n is the symbol for any stack number.

Number of cubes = $n^3 + 1$

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.

(a) What did you find when you kept on finding the differences, as suggested in question 2(a)?

The difference became constant, 6, in the third round.

(b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

The differences were consecutive odd numbers, starting from 3. Thus, the second round of differences produces a constant, 2.